

You Do
Some Big-Oh Analysis

Give a Big_Oh analysis of the running time of each function.

1. // sums the numbers from 1 to n

```
int A(int n) {  
    int sum = 0;  
    for (int i=1; i <= n; i++)  
        sum += i;  
    return sum;  
}
```

- A. $O(\log n)$
- B. $O(n)$
- C. $O(n^2)$
- D. $O(n+1)$

1.

```
int A(int n) {  
    int sum = 0;  
    for (int i=1; i <= n; i++)  
        sum += i;  
    return sum;  
}
```

Answer B: This performs n additions. $O(n)$.

Answer D: $O(n+1)$ is also correct, but we usually simplify the order of growth as much as possible.

2.

```
int B(int n) {  
    int sum = 0;  
    for (int i=1; i <= 2*n; i++)  
        for (int j=0; j < 5; j++)  
            sum += j;  
    return sum;  
}
```

Is this

- A. $O(\log n)$
- B. $O(n)$
- C. $O(n^2)$
- D. $O(5 \cdot 2 \cdot n)$

2.

```
int B(int n) {  
    int sum = 0;  
    for (int i=1; i <= 2*n; i++)  
        for (int j=0; j < 5; j++)  
            sum += j;  
    return sum;  
}
```

Analysis: The inner for-loop (on j) always adds 5 numbers together, and the outer loop (on i) does this $2*n$ times. So this is $O(5*2*n) = O(n)$. Answers B and D are both correct, but answer A is better.

3.

```
int C(int n) {  
    int sum = 0;  
    for (int i=1; i <= n; i++)  
        for (int j=0; j <= n; j++)  
            sum += j;  
    return sum;  
}
```

Is this

- A. $O(n)$
- B. $O(n^2)$
- C. $O(n^n)$
- D. The answer depends on what n is.

3.

```
int C(int n) {  
    int sum = 0;  
    for (int i=1; i <= n; i++)  
        for (int j=0; j <= n; j++)  
            sum += j;  
    return sum;  
}
```

Analysis: The inner loop (on j) runs n steps as for each value of i from 1 to n. Altogether this does $n+n+n+ \dots + n$ steps. Those numbers sum to $n*n$, so this is $O(n^2)$.

4.

```
int D(int n) {  
    int sum = 0;  
    for (int i=1; i <= n; i++)  
        sum += i*i;  
    for (int j=0; j < n; j++)  
        sum-= j;  
    for (int k = 0; k < 2*n; k++)  
        sum = sum*k;  
    return sum;  
}
```

- A. $O(n)$
- B. $O(n^2)$
- C. $O(n^3)$
- D. $O(n^n)$

4.

```
int D(int n) {  
    int sum = 0;  
    for (int i=1; i <= n; i++)  
        sum += i*i;  
    for (int j=0; j < n; j++)  
        sum -= j;  
    for (int k = 0; k < 2*n; k++)  
        sum = sum*k;  
    return sum;  
}
```

Analysis: Note that the loops are sequential, not nested. The loop on i does n additions. After that is finished the loop on j does n subtractions. Then the loop on k does $2*n$ multiplications. Altogether there are $4*n$ steps.

This is $O(n)$